

Network-wide Configuration Synthesis

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Abstract. Computer networks are hard to manage. Given a set of high-level requirements (e.g., reachability, security), operators have to manually figure out the individual configuration of potentially hundreds of devices running complex distributed protocols so that they, collectively, compute a compatible forwarding state. Not surprisingly, operators often make mistakes which lead to downtimes. Actually, the majority of network downtimes are caused by humans, not equipment failures.

We present a novel synthesis approach which automatically computes correct network configurations that comply with the operator’s requirements. Using stratified Datalog, we capture the behavior of existing routers along with the distributed protocols they run. Our key insight is to reduce the problem of finding correct input configurations to the task of synthesizing inputs for a program expressed in stratified Datalog.

To solve the synthesis task, we introduce a new iterative algorithm for stratified Datalog programs. Our algorithm partitions Datalog rules before iteratively synthesizing the inputs for each partition using off-the-shelf SMT solvers. This procedure is general and can be used to synthesize inputs for any stratified Datalog program.

We demonstrate that our approach is effective: our synthesis algorithm automatically infers correct input network configurations for a number of interesting networks running multiple widely-used distributed protocols.

1 Introduction

Despite being mission-critical for most organizations, managing a network is surprisingly hard and brittle. Starting from high-level requirements (e.g., reachability, reliability), network operators have to configure potentially hundreds of devices running complex distributed protocols so that they, collectively, compute a compatible forwarding state. Doing so requires network operators to precisely understand: *(i)* the behavior of each protocol; *(ii)* how they interact with each other; and *(iii)* how each local parameter affects the distributed computation. To complicate matters, operators often have no choice but to rely on low-level configuration interfaces that are not only vendor- but also device-dependent.

Because of this complexity, operators often make mistakes which can lead to severe network downtimes. As an illustration, Facebook (and Instagram) recently suffered from widespread issues for about an hour due to a misconfiguration [1]. In fact, studies have shown that the majority of network downtimes are caused by humans, not equipment failures [2]. Such misconfiguration can not only have local, but also Internet-wide effects. Sadly, Internet-wide outages due to misconfigurations still often make the news [3].

To prevent misconfigurations, researchers have developed tools that check if a given configuration is correct [4, 5, 6, 7]. While useful, these works still require network operators to produce the configurations in the first place. Template-based approaches [8, 9, 10, 11] along with vendor-agnostic abstractions [12, 13, 14] have been proposed to reduce the configuration burden. However, they do not abstract the underlying routing mechanisms: operators still require to precisely understand the details of each protocol. Recently, Software-Defined Networks (SDNs) have emerged as another paradigm to manage networks by *programming* them from a central controller instead of configuring them. While SDN has gained significant traction, deploying it has turned out to be a major hurdle as it requires new network devices *and* management tools. Even ignoring deployment issues, designing correct, robust and yet, scalable, SDN controllers is challenging [15, 16, 17, 18]. Because of this, only a handful of networks are using SDN in production, and only for some specific network parts (e.g., to interconnect data-centers [19, 20]). As a result, configuring individual devices is by far the most widespread (and default) way to manage networks.

The Problem: Network-Wide Configuration Synthesis. Ideally, from a network operator perspective, one would like to solve what we refer to as the *Network-Wide Configuration Synthesis* problem: *Given a network specification \mathcal{N} , which defines the behavior of all (distributed) routing protocols run by the routers, and a set \mathcal{R} of requirements on the network-wide forwarding state, discover a configuration \mathcal{C} such that the routers converge to a forwarding state compatible with \mathcal{R} .* That is, the network operator simply provides the high-level requirements \mathcal{R} , and the configuration \mathcal{C} is obtained automatically.

Distributed vs. Static routing. Relying as much as possible on distributed protocols to compute the forwarding state is critical to ensure network reliability and scalability. An alternative, but simpler, problem would be to stop using distributed protocols and statically configure the target forwarding state on each device from a central location (“à la SDN”). While static routes are sometimes useful to implement exception routing, relying solely on them is undesirable for two reasons. First, it would prevent devices from reacting locally upon failure—a key requirement in any network. Second, as the number of forwarding entries grows (large networks can easily have on the order of 100,000s entries per device), so does the cost of configuring and updating these static entries on each device. In contrast, relying on network-wide computation through configured distributed protocols enable *all* routers in the network to compute on the order of 1000 forwarding entries per second [21].

Key Challenges. Coming up with a solution to the network-wide synthesis problem is challenging for at least three reasons: *(i) Diversity:* distinct protocols exhibit different expressiveness in terms of the forwarding entries they compute. Quite often, configuring multiple protocols is required to produce a forwarding state compliant with the operators requirements \mathcal{R} . For instance, intra-domain routing protocols based on the Dijkstra algorithm (e.g., OSPF) can only compute forwarding entries which direct traffic along shortest-paths. In contrast, policy-based routing protocols such as BGP are more expressive in the

forwarding entries they compute and can direct traffic along non-shortest paths; *(ii) Dependence*: distinct protocols can depend on one another, and it is challenging to ensure that they collectively compute a compatible forwarding state satisfying \mathcal{R} . For instance, the forwarding entries computed by BGP will depend on the network-wide intra-domain configuration as BGP uses intra-domain cost to disambiguate between equivalent routes; and *(iii) Feasibility*: the search space for an input configuration is massive and it is difficult to discover the right input configuration \mathcal{C} which leads to a forwarding state satisfying the requirements \mathcal{R} .

This Work. In this paper, we address the above challenges and provide the first solution to the network-wide synthesis problem. Our approach is based on two steps. First, we use stratified Datalog to describe all network protocols, together, *once and for all*. Datalog is indeed particularly well-suited for describing these protocols in a clear and declarative way. Here, the fixed point of a Datalog program represents the forwarding state of the network.

Second, and a key insight of our work: we pose the network-wide synthesis problem as an instance of finding an input for a stratified Datalog program where the fixed point of the program satisfies a given property. That is, the network operator simply provides the high-level requirements \mathcal{R} on the forwarding state (i.e., which is the same as requiring the fixed point of the Datalog program to satisfy \mathcal{R}), and our synthesizer automatically figures out an input \mathcal{C} to the Datalog program (i.e., which is the wanted input configuration for the network).

Our Datalog synthesis algorithm is a general, independent contribution, and is applicable beyond networks. In addition, we also provide separate, network-specific optimizations which leverage our domain and speed-up the synthesis algorithm further.

Main Contributions. To summarize, our main contributions are:

- A formulation of the network-wide synthesis problem in terms of input synthesis for stratified Datalog (Section 4).
- The first input synthesis algorithm for stratified Datalog. This algorithm is of broader interest and is applicable beyond networks (Section 5).
- An instantiation of our input synthesis algorithm to the problem of network-wide synthesis, along with network-specific optimizations (Section 6).
- An implementation and validation of our approach on several networks with multiple interacting widely-used protocols (Section 7).

2 Overview

In this section, we highlight how, given a network topology and a set of network-wide requirements, our synthesizer is able to generate a network-wide configuration such that the computed forwarding state satisfies all the requirements.

Example. In the following, we consider a simple, but realistic example. We consider a network topology, depicted in Figure 1(b), composed of 4 routers denoted by A , B , C and D . Routers A and D are internal routers, while B and C are border routers connected to neighboring networks. Router D is directly

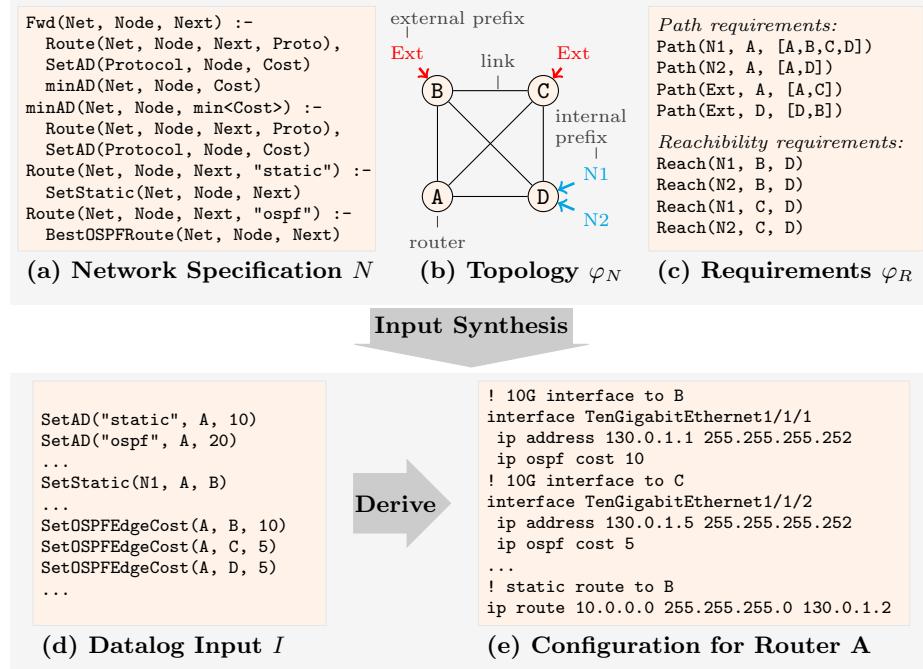


Fig. 1: Network-wide Configuration Synthesis. The input (top gray box) consists of (a) declarative network specification N that formalizes, in stratified Datalog, all protocols and their interactions, (b) network topology constraints φ_N , and (c) network-wide requirements φ_R specified as constraints over Datalog predicates. The synthesizer's output (bottom gray box) is: (d) a Datalog input I that is compatible with the network topology ($I \models \varphi_N$) and results in a forwarding state that satisfies the requirements ($\llbracket P \rrbracket_I \models \varphi_R$). Configurations (e) are derived from the Datalog input I .

connected to two internal prefixes $N1 = 10.0.0.0/24$ and $N2 = 10.0.1.0/24$ while both routers B and C can reach the external prefix $Ext = 100.0.0.0/16$.

Each router computes its own forwarding entries according to its local configuration, by running the OSPF and BGP protocols, and composing the computed OSPF and BGP routes with any static routes defined in the configuration. OSPF is an intra-domain routing protocol used by routers to compute forwarding entries directing traffic to any internal destinations along the shortest-path based on configurable link weights. In contrast, BGP is an inter-domain routing protocol used by routers to compute forwarding entries to reach external destinations based on configurable policies. In our example, the border routers B and C would advertise a route for the same prefix $100.0.0.0/16$ to A and D . Each border router is configured to associate a preference to each route. Internal routers prefers the BGP route with the highest preference. If the preference is equal, internal routers fallback to the route announced by the closest border router according to the OSPF cost to reach the router. BGP tie-breaking based on the

OSPF costs is one example of protocols interdependencies. Finally, in addition to routing using OSPF and BGP, routers can be also configured with static routes, which are statically fixed forwarding entries, as their name suggests. To instruct whether a router selects the routes computed by OSPF, BGP or those defined using static routes, each router's configuration also associate an administrative cost to each protocol. Whenever a route for a given prefix is available via more than one protocol, the router select the route associated with a lower administrative cost.

The network must satisfy four path requirements, which we depict in Figure 1(c). The first path requirement states that A must forward packets for the prefix $10.0.0.0/24$ along the path $A-B-C-D$, and the second one states that A must forward packets for $10.0.1.0/24$ directly to D . Note that these requirements cannot be enforced using the OSPF protocol alone. This is because OSPF would forward packets for both prefixes (connected to the same router, D) along the path that has the lower OSPF cost. Yet, the two requirements can be enforced by: (i) configuring a static route at A to forward packets for $10.0.0.0/24$ to B ; (ii) configuring the link weights so that the path $A-D$ has the lowest OSPF cost from A to D and the path $B-C-D$ has the lowest OSPF cost from B to D ; and (iii) configuring A to select static routes over OSPF routes by setting the administrative cost of static routes to a lower value than that of OSPF. All three configuration steps are necessary to satisfy the two requirements. The last two path requirements state that A and D must forward packets destined to the external prefix $100.0.0.0/16$ to C and B , respectively. Since both B and C would advertise this external prefix, we cannot enforce these requirements by setting different BGP preferences for B and C , because then both A and D would direct their traffic for $100.0.0.0/16$ to either B or C . Instead the two path requirements can be satisfied by: (i) setting identical BGP preference for both B and C ; and (ii) configuring the link weights so that the path $A-C$ has a lower OSPF cost than the path $A-B$, and $D-B$ has a lower OSPF cost than $D-C$.

In Figure 1(c), we also give four reachability requirements, which state that the two internal prefixes must be reachable from the routers B and C , without specifying specific paths that the packets must follow.

In the following, we illustrate how our synthesis approach is used to automatically find correct router configurations for the presented example, i.e. we describe its inputs and outputs.

2.1 Synthesizer Inputs

The input to our synthesis approach consists of (i) a declarative network specification N , expressed in stratified Datalog; (ii) network topology constraints φ_N , expressed as constraints over N 's input predicates; and (iii) requirements φ_R , expressed as constraints over N 's output predicates. The synthesizer then constructs an input I for the network specification N that precisely identifies a correct network configuration.

Declarative Network Specification (N). We formalize the network's behavior, including all routing protocols, such as OSPF and BGP, and their in-

teractions, with a stratified Datalog program N . In Figure 1(a), we show the Datalog rules that formalize how routers compute their forwarding tables, based on the OSPF routes they compute and the static routes defined in their configurations. The predicate `Fwd(Net, Node, NextHop)` defines the network’s (global) forwarding state, and it is derived if the router `Node` forwards packets destined to `Net` to the router `Next`. The predicate `Route(Net, Node, NextHop, Protocol)` is derived if the router `Node` has a path to the network `Net` using the protocol `Protocol` via the router `Next`. The first rule in the declarative specification N states that routers select, for each network, the route associated with the minimal administrative cost (`minAD`) calculated over all protocols (rule 2). Rules 3 and 4 collect the routes defined by static routes (rule 3) and OSPF (rule 4). We remark that OSPF routes, represented by the predicate `BestOSPFRoute`, are defined through additional Datalog rules that formalize OSPF; we describe our formalization of OSPF in Section 3.

Network Topology Constraints (φ_N). The network topology is expressed as constraints over the input predicates of the Datalog program N . For example, the predicate `SetLink` (not shown in Figure 1(a)) defines the links in the network.

Network-wide Requirements (φ_R). Requirements are expressed as constraints over the output predicates of the Datalog program N . These constraints can be directly defined using the predicate `Fwd`, which defines the network’s forwarding state. In this example, we consider two different types of network-wide requirements (φ_R). Path requirements are encoded using the predicate `Path(Prefix, X, PathHops)` where `PathHops` is a list of routers. They specify that traffic from `X` to `Prefix` should flow along the given path. Reachability requirements are encoded using the predicate `Reach(Prefix, X, Y)` and mandate that traffic for `Prefix` sourced by `X` should cross `Y` via any possible forwarding path. We discuss our formalization of routing requirements in Section 4.

2.2 Synthesizer Outputs

We pose the problem of network-wide configuration synthesis as an instance of finding a Datalog input I that, for the given network specification N , results in a fixed point that satisfies the requirements.

Synthesized Datalog Input I . Given the three kinds of inputs, namely N , φ_N , and φ_R described above, the synthesizer generates a Datalog input I (a set of predicates) such that the topology constraints φ_N and the routing constraints φ_R are all satisfied for the given network specification N . We depict the generated input I for our example in Figure 1(d). The input predicates contained in I identify a correct network configuration. For example, the predicate `SetAD` in Figure 1(d) sets the administrative cost for static routes to be lower than that of OSPF, which is necessary to enforce our path requirements. Further, the predicate `SetStatic(10.0.0.0/24, A, B)` sets a static route from `A` to `B` for packets destined to `10.0.0.0/24`. Finally, the predicate `SetOSPFEdgeCost` defines link weights that result in correct OSPF routes. Here, the path `A-D` has a lower cost than the path `A-B-C-D`, which is also needed to satisfy our requirements.

Derived Configurations. The synthesized Datalog input can be directly used to derive vendor-specific router configurations. For illustration, we depict an excerpt of the generated configuration for the router A in Figure 1(e).

2.3 Challenges

Posing the network configuration problem as an instance of input synthesis for stratified Datalog allows us to leverage existing Datalog systems and tools, such as [22, 23]. Unfortunately, existing systems focus on computing fixed points: given a Datalog program P , an input I , and a property φ , their focus is on computing the fixed point $\llbracket P \rrbracket_I$ for the given input and checking whether the property φ holds, i.e. $\llbracket P \rrbracket_I \models \varphi$. In contrast, to solve our problem, we start with a program P and a property φ , and then we need to compute an input I that satisfies the property.

Synthesizing inputs for stratified Datalog is, however, a difficult (and, in general, undecidable) problem [24]. To address this problem, we propose an algorithm that iteratively synthesizes the desired Datalog input. Our algorithm first partitions the Datalog program P into strata P_1, \dots, P_n . Each stratum P_i is a semi-positive Datalog program that enjoys the property that if a predicate is derived by the rules after some number of steps, then it cannot be retracted and thus must be contained in the fixed point. We leverage this property to synthesize inputs for each of the program’s strata. The algorithm iteratively synthesizes an input I_i for each stratum P_i and then constructs an input I for the Datalog program P . We describe this algorithm in Section 5.

Finally, to make our input synthesis algorithm work on practical network examples, we present network-specific optimizations, including network-specific constraints for reducing the space of possible configurations and program simplifications. We describe these in Section 6.

3 Declarative Network Specification

In this section, we first define the syntax and semantics of stratified Datalog, which is a well-studied function-free logic programming language. We also extend the language with aggregate functions (e.g., \min), which are often needed to formalize existing routing protocols. Stratified Datalog has fixed point semantics that naturally capture the iterative computation of a network’s forwarding plane performed by the routers. We illustrate how stratified Datalog can be used to declaratively specify the behaviors of networks.

3.1 Stratified Datalog

We now define the syntax and semantics of stratified Datalog.

Syntax. We define Datalog’s syntax in Figure 2(a). We use \bar{r} , \bar{l} , and \bar{t} to denote zero or more rules, literals, and terms separated by commas, respectively. A Datalog program is a set of rules of the form $a \leftarrow l_1, \dots, l_n$, where $n \geq 0$. Each rule has a *head* consisting of an atom a and a *body* l_1, \dots, l_n consisting of a (possibly empty) list of literals. An *atom* a is a predicate symbol together with

<i>(Program)</i> $P ::= \bar{r}$	<i>(Literal)</i> $l ::= a \mid \neg a$	<i>(Variables)</i> $X, Y \in Vars$
<i>(Rule)</i> $r ::= a \leftarrow \bar{l}$	<i>(Predicates)</i> $p, q \in Preds$	<i>(Values)</i> $v \in Vals$
<i>(Atom)</i> $a ::= p(\bar{t})$	<i>(Term)</i> $t ::= X \mid v$	
(a) Syntax		
<i>(Substitutions)</i> $\sigma \in Vars \rightarrow Vals$		
<i>(Ground atoms)</i> $\mathcal{A} = \{p(\bar{t}) \mid \bar{t} \subseteq Vals\}$		
<i>(Consequence operator)</i> $T_P(A) = A \cup \{\sigma(a) \mid a \leftarrow l_1 \dots l_n \in P, \forall l_i \in \bar{l}. A \vdash \sigma(l_i)\}, where$		
	$A \vdash \sigma(a)$ if $\sigma(a) \in A$ and $A \vdash \sigma(\neg a)$ if $\sigma(a) \notin A$	
<i>(Input)</i> $I \subseteq \{p(\bar{t}) \mid p(\bar{t}) \in \mathcal{A}, p \in edb(P)\}\}$		
<i>(Model)</i> $\llbracket P \rrbracket_I = M_n$, where $M_0 = I$ and $M_i = \bigcap \{A \in \text{fp } T_{P_i} \mid A \subseteq M_{i-1}\}$		
(b) Semantics for a Datalog program P with strata $P_1 \cup \dots \cup P_n$		

Fig. 2: Syntax and semantics of stratified Datalog

a list of variables and values. Values typically represent natural numbers and strings, and we assume that the set $Vals$ of values contains a bounded set of natural numbers $\{0, 1, \dots, \top\}$, where the value \top represents the largest possible natural number (here, \top is similar to the value `Integer.MAX_VALUE` in Java). Furthermore, we assume that the set $Preds$ of predicate symbols contains the comparison predicates $<$, \leq , and $=$. These predicates can be directly defined using Datalog rules. A *literal* is either an atom a (called *positive literal*) or a negated atom $\neg a$ (called *negative literal*). An atom is called *ground* if it contains no variables. Given an atom a , we write $vars(a)$ for the set of variables that appear in a , and we lift the function $vars$ to literals and list of literals in the standard way. A Datalog program is *well-formed* if for any rule $a \leftarrow \bar{l}$ in the program, we have $vars(a) \subseteq vars(\bar{l})$.

A predicate is called *extensional* if it appears only in the rules' bodies, otherwise it is called *intensional*. Given a program P , we denote its set of extensional predicates by $edb(P)$ and its set of *intensional* predicates $idb(P)$. An input for a program P is a set of ground atoms constructed using P 's extensional predicates.

A Datalog program P is *stratified* if its rules can be partitioned into sets P_1, \dots, P_n , called strata, such that (i) for every predicate symbol p , all rules with p in their heads are in one stratum P_i ; (ii) if a predicate symbol p occurs in a positive literal in P_i , then all rules with p in their heads are in a stratum P_j with $j \leq i$; (iii) if a predicate symbol p occurs in a negative literal in P_i , then all rules with p in their heads are in a stratum P_j with $j < i$.

Semantics. A substitution σ maps variables to values, and we write $\sigma(a)$ for the ground atom obtained by replacing the variables that appear in a according to σ . The semantics of a Datalog program, defined in Figure 2(b), is given by an interpretation $A \subseteq \mathcal{A}$ that contains all ground atoms derived by the program's rules. The complete lattice $(\mathcal{P}(\mathcal{A}), \subseteq, \cap, \cup, \emptyset, \mathcal{A})$ partially orders the set of all possible interpretations $\mathcal{P}(\mathcal{A})$. \subseteq partially orders the elements of $\mathcal{P}(\mathcal{A})$ where \emptyset is the least element, \mathcal{A} the greatest element, and \cap and \cup are the meet and join operators, respectively.

For a given Datalog program P , the consequence operator T_P defines which ground atoms are derived by applying P 's rules. Intuitively, given a Datalog program P and a set of (derived) ground atoms A , P derives a ground atom $\sigma(a)$ if there is a rule $a \leftarrow l_1, \dots, l_n$ in P such that for any positive literal we have $\sigma(l_i) \in A$ and for any negative literal $l_j = \neg a$ we have $\sigma(l_j) \notin A$.

Let P be a stratified Datalog program and I be an input for P . P 's model is a fixed point obtained by iteratively computing, for each stratum P_i , the smallest fixed point of T_{P_i} that is greater than the lower stratum's model M_{i-1} , where $M_0 = I$ is the program's input. Finally, P 's model, denoted by $\llbracket P \rrbracket_I$, is the fixed point M_n of the highest stratum. Stratifying P 's rules is necessary to compute P 's model because while the consequence operator T_P is non-monotone, the consequence operators T_{P_i} of P 's strata P_1, \dots, P_n are monotone.

Since network protocols often rely on aggregate functions, such as `min` and `max`, we syntactically extend stratified Datalog with common aggregate functions. Note that this extension is possible as stratified Datalog is equally expressive to Datalog with stratified aggregate functions; for details see [25]. We illustrate the encoding of such aggregate functions in Appendix A.

3.2 Specifying Networks

Stratified Datalog is sufficiently expressive to capture a variety of widely-used network protocols, their interactions, and their composition with statically defined routes. For example, OSPF, MPLS, as well as the majority of BGP, can be captured in stratified Datalog. Consequently, stratified Datalog has been used to formalize the behavior of networks with the purpose of verifying the routers' local configurations with respect to routing requirements. Researchers have even proposed to construct routers that execute distributed protocols specified directly in stratified Datalog [26]. Below, we illustrate how stratified Datalog is used to formalize networks.

A typical network consists of multiple routers that run, in de-centralized fashion, multiple distributed protocols. Each router maintains individual local configurations, one for each protocol that it executes, and those configurations influence its behavior. For example, a router running the Border Gateway Protocol (BGP) is configured with a set of policies that determines which route announcements are accepted by the router and which ones are advertised to neighboring routers. Routers individually compute a local forwarding state, which defines how incoming packets are forwarded by the router. The forwarding states of all routers then collectively define the network's (global) forwarding state. Global properties (e.g. reachability) are checked against the network's forwarding state.

To faithfully capture a network's behavior, one must model all relevant components that influence the computation of its forwarding state. This includes (*i*) the behavior of routing protocols and their interactions, (*ii*) the input protocol configurations deployed at all routers, and (*iii*) the topology of the network. Below, we show how all these components are specified in stratified Datalog.

```

BestOSPFRoute(Net, Node, NextHop) :- minCost(Net, Node, Cost),
    OSPFRoute(Net, Node, NextHop, Cost)
minCost(Net, Node, min<Cost>) :- OSPFRoute(Net, Node, NextHop, Cost)
OSPFRoute(Net, Node, Next, Cost) :- SetNetwork(_, Net),
    SetOSPFEdgeCost(Node, Next, Cost)
OSPFRoute(Net, Node, NextHop, Cost) :- Cost = Cost1 + Cost2
    SetOSPFEdgeCost(Node, NextHop, Cost1),
    OSPFRoute(Net, NextHop, Node', Cost2)

```

Fig. 3: Declarative specification of the OSPF protocol

Specifying Routing Protocols. We formalize individual routing protocols as stratified Datalog programs. We illustrate this point by formalizing the widely-used Open Shortest Path First (OSPF) protocol.

In Figure 3, we show (a subset of) of our OSPF formalization in stratified Datalog. The predicate `BestOSPFRoute(Net, Node, NextHop, Cost)` represents the best OSPF route selected by the router `Node` for the network `Net` to be the next hop `NextHop` associated with the minimum cost `Cost`. This behavior is formalized with the first rule in Figure 3. The second rule derives the minimum cost OSPF route for each router and each destination network by aggregating over all possible OSPF routes. Finally, the last two rules concisely implement the shortest-path computation performed by the routers running OSPF. The predicate `SetOSPFEdgeCost(Node1, Node2, Cost)` represents that the routers `Node1` and `Node2` are neighbors connected by a link with cost `Cost`, and the predicate `SetNetwork` for any value that represents a network. The third rule thus formalizes that `Node1` can forward packets to `Node2`, for any network `Net`, with this cost. The last rule transitively computes multi-hop routing paths by summing up the costs associated along all OSPF routes.

Routing protocols often have interdependencies, i.e., the computation of one routing protocol may be given as input to another routing protocol executed by the same router. For example, the selected best OSPF routes are provided as input to the BGP protocol. Further, the routes computed by OSPF, BGP, and statically defined routes, are all composed based on the administrative costs configured in the router, as we have illustrated in the example of Section 2. Such interdependencies are directly captured by simply taking the union of all Datalog programs that specify the individual protocols and their composition. For example, the Datalog specification of BGP would then take as input the `BestOSPFRoute` predicate, which is, in turn, derived by the Datalog rules that formalize the OSPF protocol (see Figure 3).

Network-wide Configurations. The input protocol configurations deployed at the network's routers are represented as input `edb` predicates to the Datalog programs that formalize these protocols. In OSPF, for example, the local OSPF configuration for a given router specifies the costs associated with the router's immediate OSPF neighbors. This is represented by the predicate `SetOSPFEdgeCost(Node, NextHop, Cost)` in Figure 3. Note that this predicate represents the OSPF configurations deployed at all routers. The local configuration for a given router, e.g. "R1", is obtained by projecting the predicate on the vari-

able `Node`, namely `SetOSPFNeighbor("R1", NextHop, Cost)`. We refer to the set of all local router configurations as the *network-wide* configuration.

Network Topology. Formalizing the topology is needed to capture the network’s behavior, as many routing protocols are influenced by how the routers are interconnected. We formalize the network topology using input *edb* predicates for the declarative network specification. We model each router and each interface as a constant, and we use the predicate `SetInterface(Node, IFace)` to specify the interfaces associated with each router, and `SetLink(IFace1, IFace2)` to model that two interfaces are connected by a link.

4 Network-wide Configuration Synthesis

We now formalize the network-wide configuration problem as a synthesis problem that takes as input a formal description of the network in stratified Datalog, a network topology, and a set of global requirements, and outputs a network-wide configuration, which defines the individual configurations to be deployed at all routers, so that all requirements are met. Additionally, the synthesis problem takes as input a set of configuration constraints (not shown in Figure 1), which are necessary to formalize which configurations are well-formed. Below, we first define a simple constraint language, and then show how it can be used to formalize (i) global routing requirements, (ii) the network topology, and (iii) the configuration constraints. We conclude the section with a formal definition of the network-wide configuration synthesis problem.

4.1 Constraints

We define a simple language to express function-free first-order constraints defined over the same signature used to formalize the network in stratified Datalog.

Syntax. A constraint φ is a formula of the form

$$\varphi = \text{true} \mid p(\bar{t}) \mid \neg\varphi \mid \varphi \wedge \varphi \mid \forall X. \varphi$$

where $p \in \text{Preds}$ is a predicate and $X \in \text{Vars}$ is a variable. Additional connectives such as disjunction \vee , implication \Rightarrow , and the existential quantifier \exists can be defined in the standard way; e.g., $\exists X. \varphi$ is defined as $\neg(\forall X. (\neg\varphi))$. Given a constraint φ , we write $\text{fv}(\varphi)$ for the set of variables in φ that are not in the scope of \forall . A constraint φ is well-formed if $\text{fv}(\varphi) = \emptyset$.

Semantics. A Datalog interpretation A naturally induces a satisfaction relation \models between interpretations and constraints and we inductively define this satisfaction relation as follows:

$$\begin{array}{ll} A \models \text{true} & A \models p(\bar{t}) \quad \text{if } p(\bar{t}) \in A \\ A \models \neg\varphi & \text{if } A \not\models \varphi \\ A \models \forall X. \varphi & \text{if } \forall v \in \text{Vals}. A \models \varphi[X/v] \end{array} \quad \begin{array}{ll} A \models p_1 \wedge p_2 & \text{if } A \models p_1 \text{ and } A \models p_2 \end{array}$$

Here, $\varphi[X/v]$ denotes that all occurrences of the variable X in the constraint φ are replaced with the value v .

4.2 Network-wide Routing Requirements

The forwarding state of the network must conform to network-wide routing requirements. Examples include *path* requirements, such as “packets for network \mathbb{N}

follow the routing path A, B, C , and D ”, *reachability* requirements, such as “packets for network N can reach router $R1$ from router $R2$ ”, *waypointing* constraints, such as “all packets for network N that go from $R1$ to $R2$ must pass through the firewall FW ”, and *generic* requirements, such as the absence of black holes.

To formulate such routing requirements, we fix the predicate `Fwd(Net, Node, NextHop)` to denote the (global) forwarding state of the network. Intuitively, `Fwd(Net, Node, NextHop)` is derived if the router `Node` forwards packets destined to the network `Net` to the router `NextHop`. Further, we assume the following predicates are defined, through Datalog rules, in the network specification: `Reach(Net, Src, Dst)`, which is derived if packets destined to the network `Net` have a forwarding path from `Src` to `Dst`, and `Waypoint(Net, Src, Mid, Dst)`, which is derived if packets destined to `Net` that go from `Src` to `Dst` pass through `Mid`.

Network-wide routing requirements are then formalized as constraints over the predicates `Fwd`, `Reach`, and `Waypoint`. The predicate `Fwd` can be directly used to specify path requirements, which stipulate that packets for a given network must follow a specific routing path. This is specified by taking the conjunction of the predicates `Fwd` that define the path. For example,

$$\text{Fwd}(N1, R1, R2) \wedge \text{Fwd}(N1, R2, R3)$$

specifies that packets for network `N1` follow the routing path `R1, R2`, and `R3`. We will write `Path(Net, R1, [R1, R2, ..., Rn])` as a shorthand for the conjunction `Fwd(Net, R1, R2) \wedge \dots \wedge Fwd(Net, Rn-1, Rn)` when specifying path requirements. The predicates `Reach` and `Waypoint` are used to encode reachability and waypointing requirements, respectively. The absence of black holes and forwarding loops are easily specified using these predicates as well. Concretely, the constraint

$$\forall Net, R. (\neg \text{Reach}(Net, R, R))$$

specifies that there must be no loops in the forwarding plane. The constraint

$$\exists Net, R1, R2. \text{Fwd}(Net, R1, R2) \wedge (\neg \text{Reach}(Net, R2, Next))$$

holds iff there is a black hole in the network. Namely, it holds iff there is a router `R1` that forwards packets destined to some network `Net` to router `R2`, but `R2` has no path to `Net` along which it can forward `R1`'s packets. The absence of black holes is thus specified by taking the negation of this constraint.

4.3 Network Topology

This is defined as constraints over input *edb* predicates such as `SetNetwork(Node, Net)`, `SetInterface(Node, IFace)`, and `SetLink(IFace1, IFace2)`, which have intuitive meaning. To set these predicates, suppose the links set in the network are $(I1, I2)$ and $(I3, I4)$. We fix that precisely these two links are set in the predicate `SetLink` with the constraint:

$$\forall X, Y. ((X = I1 \wedge Y = I2) \vee (X = I3 \wedge Y = I4)) \Leftrightarrow \text{SetLink}(X, Y)$$

This constraint formalizes that the only `SetLink` predicates that are true are `SetLink(I1, I2)` and `SetLink(I3, I4)`.

4.4 Configuration Constraints

The configurations provided as inputs to network protocols must satisfy constraints that are defined in the protocol specifications. We restrict the protocol configuration constraints to be specified as constraints over extensional predicates, as they constrain the protocols' inputs, not facts derived by the protocols. For example, the costs defined in an OSPF configuration, defined by the *edb* predicate `SetOSPFNeighbor(Node, NextHop, Cost)`, see Figure 3, must be positive integers. We can formalize this protocol configuration constraint as:

$$\forall \text{Node}, \text{NextHop}, \text{Cost}. (\text{SetOSPFNeighbor}(\text{Node}, \text{NextHop}, \text{Cost}) \Rightarrow \text{Cost} > 0)$$

4.5 Network-wide Configuration Synthesis Problem

We now formally define the network-wide configuration synthesis problem.

Definition 1. *The network configuration synthesis problem is:*

- Input** A network specification N , global requirements φ_R , network topology φ_N , and a protocol configuration constraints φ_C .
Output An input I such that $I \models \varphi_N \wedge \varphi_C$ and $\llbracket N \rrbracket_I \models \varphi_R$, if such an input exists, and otherwise it returns *unsat*.

Recall that the input I identifies the configurations for all routers, as described in Section 3.2. The conditions on the synthesized output guarantee that: (i) the protocol configurations are well-formed since $I \models \varphi_C$, (ii) they are compatible with the network topology since $I \models \varphi_N$, and (iii) they results in a correct forwarding state since $\llbracket N \rrbracket_I \models \varphi_R$.

We remark on several key points on the network synthesis problem. First, for any *edb* atom a , we have $a \in I$ if and only if $a \in \llbracket N \rrbracket_I$. Therefore, $\llbracket N \rrbracket_I \models \varphi_N \wedge \varphi_C$ implies that $I \models \varphi_N \wedge \varphi_C$. To solve the network synthesis problem, it is thus equivalent to find an input I such that $\llbracket N \rrbracket_I \models \varphi_R \wedge \varphi_N \wedge \varphi_C$. Second, the conjunction $\varphi_R \wedge \varphi_N \wedge \varphi_C$ can be encoded through Datalog rules, such that a designated atom a_{SAT} is derived if and only if $\llbracket N \rrbracket_I \models \varphi_R \wedge \varphi_N \wedge \varphi_C$. The satisfiability of the network synthesis problem can be checked by solving an instance of the query satisfiability problem in stratified Datalog, which asks whether for a given stratified Datalog program N and a ground atom a , there exists an input I such that $a \in \llbracket N \rrbracket_I$ (and vice versa, a query satisfiability problem can be answered by solving an instance of the network synthesis problem). Since the query satisfiability problem in stratified Datalog is, in general, undecidable [24], the network synthesis problem is undecidable as well.

The problem is, however, decidable if we fix a finite set of values, which yields a finite number of possible inputs. In the context of networks, it is reasonable to bound the number of configurations considered for synthesis, as the set of values are used to represent finitely many routers, interfaces, and configuration parameters.

5 Input Synthesis for Stratified Datalog

In this section, we present a new iterative algorithm for synthesizing inputs for stratified Datalog. We believe this algorithm is of general interest, beyond

networks. We return to networks in Section 6 and show how the algorithm of this section is used to solve the network-wide configuration synthesis problem.

Input Synthesis Problem for Stratified Datalog. We start by formalizing the problem of input synthesis for stratified Datalog.

Definition 2. *The input synthesis for stratified Datalog is:*

Input A stratified Datalog program P and a constraint φ .

Output An input I such that $\llbracket P \rrbracket_I \models \varphi$, if such an input exists, otherwise, return *unsat*.

We first define an algorithm, called $\mathcal{S}_{SemiPos}$, that addresses the above problem for the semi-positive Datalog fragment, which contains all stratified Datalog programs where all negative literals are constructed using *edb* predicate symbols. By definition of stratification, each stratum of a stratified Datalog program is a semi-positive Datalog program. We then present an algorithm that leverages the $\mathcal{S}_{SemiPos}$ algorithm to synthesize inputs for any stratified Datalog program.

5.1 Input Synthesis for Semi-positive Datalog with SMT

The key idea is to reduce the problem of input synthesis to satisfiability of a set of SMT constraints. That is, for a given semi-positive Datalog program P and a constraint φ , we encode the question $\exists I. \llbracket P \rrbracket_I \models \varphi$ into SMT constraints ψ such that if the constraints ψ are satisfiable, then the question is answered positively. We can then use a model of ψ to derive an input I such that $\llbracket P \rrbracket_I \models \varphi$.

SMT Encoding Each non-recursive *idb* predicate can be directly encoded by taking the disjunction over all rules' bodies that have the *idb* predicate in their heads. The encoding of recursive *idb* predicates, such as the reachability predicate `Reach(Net, Src, Dst)`, is however non-trivial due to the mismatch between Datalog's least fixed-point semantics and the classical semantics of first-order logic; see Appendix B for a concrete illustration of the problem.

Our encoding of recursive *idb* predicates is based on the following two key insights. Our first insight is to split the constraint φ into a conjunction of positive and negative clauses. Formally, let A be an interpretation and φ a constraint. The constraint φ is positive (respectively, negative) if $A \models \varphi$ implies that $A' \models \varphi$ for any $A' \supseteq A$ (respectively, for any $A' \subseteq A$). Our second insight is to use a different encoding for both positive and negative constraints. Namely, we unroll recursive predicates to obtain a sound encoding for positive constraints, and we do not unroll them to get a sound encoding for negative requirements.

We define the translation of a Datalog program P into SMT constraints in Figure 4. The encoding is parameterized by a parameter $k \geq 0$, which defines the number of times we unroll predicates. The resulting SMT constraint is denoted by $[P]_k$. For simplicity, in the translation rules we assume that (i) all terms that appear in the rules' heads are variables and (ii) for any two atoms that appear in the rules' heads, if they have the same predicate then they also have an identical list of variable names. Programs that do not satisfy these assumptions can be converted into such form by rectification [27] and variable renaming.

$$\begin{aligned}
[P]_k &= \bigwedge_{p \in idb(P)} \text{ENCODE}(P, p) \wedge \text{UNROLL}(P, p, k) \\
\text{ENCODE}(P, p) &= \bigwedge_{p(\bar{X}) \leftarrow \bar{l} \in P} \forall \bar{X}. ((\exists \bar{Y}. \bigwedge \bar{l} \Rightarrow p(\bar{X})), \text{where } \bar{Y} = vars(\bar{l}) \setminus \bar{X}) \\
\text{UNROLL}(P, p, k) &= \bigwedge_{0 \leq i \leq k} \text{STEP}(P, p, i) \\
\text{STEP}(P, p, i) &= \forall \bar{X}. (p_i(\bar{X}) \Leftrightarrow (\bigvee_{p(\bar{X}) \leftarrow \bar{l} \in P} \exists \bar{Y}. \tau(\bar{l}, i - 1))), \text{where } \bar{Y} = vars(\bar{l}) \setminus \bar{X} \\
\tau(\bar{l}, k) &= \begin{cases} \tau(l_1, k) \wedge \dots \wedge \tau(l_n, k) & \text{if } \bar{l} = l_1 \wedge \dots \wedge l_n \\ \neg \tau(p(\bar{l}), k) & \text{if } \bar{l} = \neg p(\bar{l}) \\ \text{false} & \text{if } \bar{l} = p(\bar{l}), p \in idb(P), k = 0 \\ p_k(\bar{l}) & \text{if } \bar{l} = p(\bar{l}), p \in idb(P), k > 0 \\ l & \text{otherwise} \end{cases}
\end{aligned}$$

Fig. 4: Encoding a Datalog program P with constraints $[P]_k$

The Translation Function Encode. We now describe the translation of a Datalog program P into SMT constraints $[P]_k$. Given an idb predicate symbol $p \in idb(P)$, the SMT constraint returned by $\text{ENCODE}(p, P)$ states that an atom $p(X)$ is derived if there is a rule in P that has $p(\bar{X})$ in the head and its body evaluates to true. The variables \bar{X} that appear in the head $p(\bar{X})$ are universally quantified, while the variables in the rules' bodies are existentially quantified, as is standard in Datalog's semantics. Note that if the atom $p(\bar{l})$ has cyclic-dependent predicates, i.e., $p(X)$ is derived from predicates that are, in turn, derived using $p(X)$, then such cyclic-dependent predicates appear on the left-hand side of the implication (\Rightarrow). The SMT constraint output by ENCODE is sound for negative requirements. It is however unsound for positive ones as it does not formalize that a head atom $p(\bar{X})$ is derived only if a rule body with $p(\bar{X})$ in the head evaluates to true.

The Translation Functions Unroll and Step. The translation functions UNROLL and STEP rely on the literal renaming function τ , which takes a list of literals and a positive integer and returns either a renamed version of the literals or false . Intuitively, for a given literal l , we use $\tau(l, k)$ to encode whether the literal l is true, or false, after k derivation steps. For example, the truth value of $p(\bar{l})$ after 3 steps is represented with the atom $p_3(\bar{l})$. Since semantics of all intensional predicates are false before P 's rules have been applied, $\tau(l, 0)$ returns false , for any intensional literal l . Further, since extensional predicates do not change their values, $\tau(l, k)$ returns l for any extensional literal l (the case “otherwise” in Figure 4).

Given an atom a and a program P , the SMT constraint $\text{UNROLL}(a, P, k)$ encodes the truth value of an atom a after k applications of P 's rules. The truth value of the atom a after the i th derivation step, in turn, is encoded using the translation function $\text{STEP}(a, P, i)$. Intuitively, the atom a is true after the i th derivation step if and only if there is a rule with a in the head such that its body evaluates to true using the atoms derived in previous iterations. Since during the fixed point calculation of Datalog, the i th step uses the atoms derived in the previous iterations, we take the truth values of $\tau(l, i - 1)$ for all literals l that

Algorithm 1: The input synthesis algorithm $S_{SemiPos}$ for semi-positive Datalog programs

Input: Semi-positive Datalog program P and a constraint φ
Output: An input I such that $\llbracket P \rrbracket_I \models \varphi$ or \perp

```

1 begin
2    $\varphi' \leftarrow \text{SIMPLIFY}(\varphi)$ 
3   for  $k \in [0..bound_k]$  do
4      $\varphi_k \leftarrow \text{REWRITE}(\varphi', k)$ 
5      $\psi \leftarrow [P]_k \wedge \varphi_k$ 
6     if  $\exists J. J \models \psi$  then
7        $I \leftarrow \{p(\bar{t}) \in J \mid p \in \text{edb}(P)\}$ , where  $J \models \psi$ 
8       return  $I$ 
9   return  $\perp$ 

```

appear in the rules' bodies. Note that the rules which are output by UNROLL have no cyclic dependencies. This guarantees that the encoding is sound for positive requirements.

Example. To illustrate the encoding, we translate the Datalog program with rules:

$$\begin{aligned} tc(X, Y) &\leftarrow e(X, Y) \\ tc(X, Y) &\leftarrow tc(X, Z), tc(Z, Y) \end{aligned}$$

which computes the transitive closure of the predicate $e(X, Y)$. The only intensional atom derived by P 's rules is $tc(X, Y)$. The function $\text{ENCODE}(a, P)$ returns the constraint

$$\begin{aligned} &(\forall X, Y. e(X, Y) \Rightarrow tc(X, Y)) \\ &\wedge (\forall X, Y. (\exists Z. tc(X, Z) \wedge tc(Z, Y)) \Rightarrow tc(X, Y)) \end{aligned}$$

We apply function $\text{UNROLL}(a, P, 2)$ for $k = 2$, which after simplifications returns

$$\begin{aligned} &\forall X, Y. (tc_1(X, Y) \Leftrightarrow e(X, Y)) \\ &\forall X, Y. (tc_2(X, Y) \Leftrightarrow e(X, Y) \vee (\exists Z. tc_1(X, Z) \wedge tc_1(Z, Y))) \end{aligned}$$

In the constraints, the predicates tc_1 and tc_2 encode the derived predicates tc after 1 and, respectively, 2, derivation steps.

Algorithm. Our algorithm uses the helper function $\text{REWRITE}(\varphi', k)$:

$$\text{REWRITE}(\varphi, k) = \begin{cases} p_k(\bar{t}) & \text{if } \varphi = p(\bar{t}) \\ \neg p(\bar{t}) & \text{if } \varphi = \neg p(\bar{t}) \\ \text{REWRITE}(\varphi_1, k) \vee \dots \vee \text{REWRITE}(\varphi_n, k) & \text{if } \varphi = \varphi_1 \vee \dots \vee \varphi_n \\ \text{REWRITE}(\varphi_1, k) \wedge \dots \wedge \text{REWRITE}(\varphi_n, k) & \text{if } \varphi = \varphi_1 \wedge \dots \wedge \varphi_n \end{cases}$$

This function takes as input a constraint φ and a parameter k and it recursively traverses all conjunctions and disjunctions. Note that if the constraint φ is in conjunctive normal form, since the operators \vee and \wedge are monotone, all negative literals constitute negative constraints while all positive literals in φ constitute positive constraints. The function $\text{REWRITE}(\varphi, k)$ therefore maps positive literals to the k -unrolled predicate $p_k(\bar{t})$ and negative literals to $\neg p(\bar{t})$.

The algorithm for synthesizing inputs for semi-positive Datalog programs is given in Algorithm 1. It takes as input a semi-positive Datalog program P and a constraint φ . First, the algorithm simplifies the constraint φ into conjunctive normal form. The resulting constraint, φ' in the algorithm, is a conjunction of clauses, where each clause is a disjunction of literals. The algorithm then iteratively unrolls the Datalog rules, up to a pre-defined bound, called $bound_k$. In each step of the for-loop, the algorithm first rewrites the constraints using the function REWRITE and checks its satisfiability. If it is satisfiable, then an input is derived by projecting the interpretation I that satisfies the constraint over all edb predicates.

We conclude the algorithm by stating its correctness.

Theorem 1. *Let P be a semi-positive Datalog program, φ a constraint. If $\mathcal{S}_{SemiPos}(P, \varphi) = I$ then $\llbracket P \rrbracket_I \models \varphi$.*

We remark that for any semi-positive Datalog program P and input I , the model $\llbracket P \rrbracket_I$ is computed in finitely many steps. If there exists an input I such that $\llbracket P \rrbracket \models \varphi$, for a given constraint φ , that reaches a fixed point in less steps than the $bound_k$ used in the algorithm, then the algorithm is also guaranteed to find an input I' such that $\llbracket P \rrbracket_{I'} \models \varphi$.

5.2 Iterative Input Synthesis for Stratified Datalog

We now present an iterative input synthesis algorithm for stratified Datalog. This algorithm uses the $\mathcal{S}_{SemiPos}$ algorithm of Section 5.1 as a subroutine.

Algorithm. Algorithm 2 details the main steps of the iterative input synthesis algorithm for stratified Datalog. The algorithm takes as input a stratified Datalog program P and a constraint φ . To simplify our algorithm, we assume that the constraint φ is defined only over the predicates that appear in P_n . This is without any loss of generality, as any constraints over the predicates that appear in the lower strata can be expressed using Datalog rules in the highest stratum, via a standard reduction to query satisfiability; see Appendix C.

High Level Flow. At a high-level, the algorithm starts with the highest stratum P_n , and generates an input I_n for P_n such that $\llbracket P_n \rrbracket_{I_n} \models \varphi$. Then, the algorithm iteratively synthesizes an input for each stratum P_{n-1}, \dots, P_1 , starting with the highest stratum and going towards the lower ones, until it generates inputs for all strata. In Datalog’s semantics, the output of a stratum P_i , i.e. the fixed point of P_i , is given as input to the higher stratum P_{i+1} . While synthesizing the strata inputs, the synthesis algorithm ensures that the synthesized input I_i for the stratum P_i produces a fixed point $\llbracket P_i \rrbracket_{I_i}$ that is “compatible” with the input generated for I_{i+1} ; formally, $\{p(\bar{t}) \in \llbracket P_i \rrbracket_{I_i} \mid p \in edb(P_{j>i})\} = \{p(\bar{t}) \in I_{i+1} \mid p \in edb(P_{j>i})\}$. Finally, to construct the input I for P , the algorithm collects all atoms constructed with predicates in $edb(P)$ and returns this input.

The Helper Function EncodePred. The function $ENCODEPRED(I, p)$ in the algorithm returns a constraint that fixes all atoms constructed with the

Algorithm 2: Input synthesis algorithm \mathcal{S}_{Strat} for stratified Datalog

Input: Stratified Datalog program $P = P_1 \cup \dots \cup P_n$, constraint φ over P_n
Output: An input I such that $\llbracket P \rrbracket_I \models \varphi$ or \perp

```

1 begin
2    $\mathcal{F}_1 \leftarrow \emptyset, \dots, \mathcal{F}_n \leftarrow \emptyset; I_1 \leftarrow \perp, \dots, I_n \leftarrow \perp; i \leftarrow n$ 
3   while  $i > 0$  do
4     if  $|\mathcal{F}_i| > bound_{\mathcal{F}}$  then
5        $\mathcal{F}_i \leftarrow \emptyset; \mathcal{F}_{i+1} \leftarrow \mathcal{F}_{i+1} \cup \{I_{i+1}\}; i \leftarrow i + 1$ ; continue
6      $\psi_{\mathcal{F}} \leftarrow \bigwedge_{I' \in \mathcal{F}_i} (\neg \bigwedge_{p \in edb(P_i)} \text{ENCODEPRED}(I', p))$ 
7     if  $i = n$  then
8        $\psi_i \leftarrow \varphi$ 
9     else
10       $\psi_i \leftarrow \bigwedge_{p \in edb(P_i) \cup idb(P_i)} \text{ENCODEPRED}(I_{i+1} \cup \dots \cup I_n, p)$ 
11       $I_i = \mathcal{S}_{SemiPos}(P_i, \varphi_i \wedge \psi_{\mathcal{F}})$ 
12      if  $I_i \neq \perp$  then
13         $i \leftarrow i - 1$ 
14      else
15        if  $i = n$  then
16          return  $\perp$ 
17        else
18           $\mathcal{F}_i \leftarrow \emptyset; \mathcal{F}_{i+1} \leftarrow \mathcal{F}_{i+1} \cup \{I_{i+1}\}; i \leftarrow i + 1$ 
19
20    $I = \{p(\bar{t}) \in I_1 \cup \dots \cup I_n \mid p \in edb(P)\}$ 
21   return  $I$ 

```

predicate p according to the interpretation I . Formally,

$$\begin{aligned} \text{ENCODEPRED}(I, p) = & ((\bigvee \{\bar{X} = \bar{t} \mid p(\bar{t}) \in I\}) \Rightarrow p(\bar{X})) \\ & \wedge \\ & ((\neg(\bigvee \{\bar{X} = \bar{t} \mid p(\bar{t}) \in I\})) \Rightarrow \neg p(\bar{X})) \end{aligned}$$

The first disjunction specifies that if an atom $p(\bar{t})$ is contained in I , then it must be derived, and the second disjunction states that if an atom $p(\bar{t})$ is not contained in I , then it must not be derived, namely $\neg p(\bar{X})$. Consider a stratum P_i , a predicate $p \in edb(P_i) \cup idb(P_i)$, and an interpretation I for P_i . Let $\psi = \text{ENCODEPRED}(I, p)$. For any interpretation I' such that $I' \models \psi$ and any atom $p(\bar{t})$, we have $p(\bar{t}) \in I'$ iff $p(\bar{t}) \in I$. That is, interpretations that satisfy ψ have the same sets of atoms constructed using the predicate p as those contained in I . The algorithm uses such constraints to ensure that each lower strata's fixed point is compatible with the already synthesized inputs for the higher strata, as we explain below.

Key Steps. We now describe the key steps of \mathcal{S}_{Strat} . The rules in P are first partitioned into strata P_1, \dots, P_n . Such a partitioning is easily obtained using the predicates' dependency graph, see [28, Chapter 15.2] for a concrete algorithm.

For each stratum P_i , the algorithm maintains a set \mathcal{F}_i of inputs. An input is added to \mathcal{F}_i , if the algorithm fails to generate inputs for the lower strata P_1, \dots, P_{i-1} that are compatible with I . We refer to the inputs in the sets \mathcal{F}_i as *failed* sets of inputs. Initially, all sets \mathcal{F}_i are set to the empty set (line 2), and all strata inputs I_1, \dots, I_n are set to \perp (line 2). In each iteration of the while loop, with $i \in [1..n]$, the algorithm attempts to generate an input for the stratum P_i .

At line 4, the algorithm checks whether the set of failed inputs \mathcal{F}_i for stratum P_i has exceeded a pre-defined bound $bound_{\mathcal{F}}$. This is used to avoid exhaustively searching through all inputs to find an input compatible with those synthesized for the higher strata. If the bound is exceeded, the algorithm backtracks to a higher stratum (by incrementing i). Before moving to the higher stratum, the algorithm adds the input I_{i+1} to the set of failed inputs \mathcal{F}_{i+1} and re-initializes its set of failed inputs \mathcal{F}_i to the empty set.

At line 6 the algorithm constructs a constraint $\psi_{\mathcal{F}}$, which is satisfied for some input I_i if and only if $I_i \notin \mathcal{F}_i$. The algorithm uses $\psi_{\mathcal{F}}$ to avoid generating inputs for which the lower strata failed to generate inputs. The constraint ψ_i constrains the output of the stratum P_i , i.e. its *edb* predicates. For the highest stratum P_n , the constraint ψ_i , constructed at line 8, is set to the constraint φ provided as input to the algorithm. For the remaining strata P_i , the constraint ψ_i , constructed at line 10, is satisfied if and only if the output of P_i is compatible with the synthesized inputs for the higher strata P_{i+1}, \dots, P_n . In addition to constraining P_i 's output, i.e. its *edb* predicates, we also constraint the input *edb* predicates. This is necessary to eagerly constraint the inputs, as detail shortly.

At line 11, the algorithm invokes the $\mathcal{S}_{SemiPos}$ algorithm to generate an input I_i such that $\llbracket P_i \rrbracket_{I_i} \models \varphi_i \wedge \psi_{\mathcal{F}}$. If such an input is found, i.e., $I_i \neq \perp$, then the synthesis algorithm proceeds to synthesize an input for the lower stratum (by decrementing i). Otherwise, the algorithm either returns \perp if it failed to generate an input for the highest stratum (line 16) or it backtracks to the higher stratum by increasing i and updating the sets \mathcal{F}_{i+1} and \mathcal{F}_i .

Finally, the while-loop terminates when the inputs of all strata have been generated. The algorithm constructs and returns the input I for P .

Theorem 2. *Let P be a stratified Datalog program with strata P_1, \dots, P_n , and φ a constraint over predicates in P_n . If $\mathcal{S}_{Strat}(P, \varphi) = I$ then $\llbracket P \rrbracket_I \models \varphi$.*

We remark that the algorithm \mathcal{S}_{Strat} eagerly constraints the input synthesized for each stratum P_i . This is necessary to avoid synthesizing inputs I_i for which the lower strata P_0, \dots, P_{i-1} do not have inputs that are compatible with I_i . We give an example in Appendix D. Furthermore, we partially evaluate the Datalog rules with respect to the synthesized inputs, before translating them into SMT, to reduce the number of variables in the Datalog rules, which in turn reduces the variables in the generated SMT constraints.

6 Network Synthesis Algorithm

We now show how we use the algorithm \mathcal{S}_{Strat} to synthesize protocol configurations for networks.

Algorithm 3: The algorithm \mathcal{S}_{Net} for synthesizing correct network-wide configurations.

Input: Network specification N , global requirements φ_R , network topology φ_N , and protocol configuration constraints φ_C

Output: An input I such that $I \models \varphi_N \wedge \varphi_C$ and $\llbracket N \rrbracket_I \models \varphi_R$, or \perp

```

1 begin
2   Compute  $Q$  such that  $\varphi_N \wedge \varphi_C \wedge \varphi_R \xrightarrow{a_\vee} Q$ 
3   Stratify  $N \cup Q$  into partitions  $N_1, \dots, N_n$  such that  $a_\vee \in idb(N_n)$ 
4    $I \leftarrow \mathcal{S}_{Strat}(N \cup Q, a_\vee)$ 
5   return  $I$ 

```

6.1 Network Synthesis Algorithm

The main steps of our algorithm for synthesizing network configurations is given in Algorithm 3. Given a network specification N , requirements φ_R , network topology φ_N , and configuration constraints φ_C , we need to generate an input I for N such that $I \models \varphi_N \wedge \varphi_C$ and $\llbracket N \rrbracket_I \models \varphi_R$. This is equivalent to synthesizing an input I such that $\llbracket N \rrbracket_I \models \varphi_N \wedge \varphi_C \wedge \varphi_R$. Since \mathcal{S}_{Strat} requires that the input constraint is defined over predicates in N 's highest stratum, and φ_N and φ_C may refer to predicates in lower strata, as a first step we translate the constraint $\varphi_N \wedge \varphi_C \wedge \varphi_R$ into a set Q of Datalog rules that contains a designated predicate a_\vee such that for any input I for Q , we have $\llbracket Q \rrbracket_I \models \varphi_N \wedge \varphi_C \wedge \varphi_R$ if and only if $a_\vee \in \llbracket Q \rrbracket_I$. This translation is denoted by $\xrightarrow{a_\vee}$ in Algorithm 3. We remark that the translation $\xrightarrow{a_\vee}$ is analogous to a standard query satisfiability reduction for Datalog; for details see Appendix C.

Second, the algorithm extend the network specification N with the set Q of Datalog rules, which is obtained after translating $\varphi_R \wedge \varphi_N \wedge \varphi_C$, and stratifies the rules in $N \cup Q$ into strata N_1, \dots, N_n . Note that since the atom a_\vee does not appear in the body of a rule in $N \cup Q$, the rule with the head a_\vee can be placed in the highest stratum N_n . Note that $edb(N) = edb(N \cup Q)$, and therefore extending N with Q does not change the signature of N 's inputs.

Finally, we invoke the algorithm \mathcal{S}_{Strat} for the inputs $N \cup Q$ and the constraint a_\vee and the algorithm returns the answer output by the algorithm \mathcal{S}_{Strat} .

6.2 Network-specific Optimizations

We now describe several optimizations that are applied to declarative network specifications, before they are given to the algorithm \mathcal{S}_{Strat} . Such optimizations are key in making the synthesis algorithm applicable to practical network configuration synthesis problems.

Network Topology Specialization. The network topology is encoded as a set of *edb* predicates, such as `SetLink`, which are provided as input to the formal network specification N . As all such predicates are known apriori, we can eliminate them from the Datalog rules using partial evaluation on the stratified Datalog program N .

Network-wide Constraints. As a second optimization, we introduce network-specific constraints. Intuitively, these capture generic properties that are true for all forwarding states, as well as protocol-specific properties, which are constraints that hold for any input to a particular protocol. An example of a network-specific constraint is: “*No packet is forwarded out of the router if the destination network is directly connected to the router*. We remark that these constraints are neither specific to a particular set of requirements nor to a specific network topology. They are therefore defined one time for each protocol and can be used to synthesize configurations for any requirements and any network that use the particular routing protocol.

7 Implementation and Evaluation

We next describe an implementation and evaluation of our algorithm on three case studies which capture practical topologies and requirements. Our results highlight the feasibility of network-wide configuration synthesis.

7.1 Implementation

We implemented the synthesis algorithm presented in Section 6 in Python. As part of the implementation of the $\mathcal{S}_{SemiPos}$ algorithm, we automated the translation from Datalog rules, which are specified in the LogicBlox language [29], into SMT constraints specified in the generic SMT-LIB v2 format [30]. Our implementation uses the Python API of the Z3 SMT solver [31] to check whether the generated SMT constraints are satisfiable and to obtain a model.

Our system supports networks with routers running OSPF and BGP along with static routes. The composition of the forwarding routes computed by OSPF and BGP, and those defined using static routes is formalized with additional Datalog rules. We merged all Datalog rules that formalize the supported protocols and their composition, and we stratified the resulting network specification into 8 partitions. We also defined additional SMT constraints that ensure the well-formedness of the OSPF and static routes configurations output by our synthesizer. To reduce the space of possible configurations, we employed the network-specific constraints, as described in Section 6.2.

7.2 Experiments

To investigate the feasibility and scalability of our approach, we experimented with different: (*i*) network topologies and network requirements; along with (*ii*) different protocol combinations.

Network Topologies and Path Requirements. We used network topologies with 4, 9, and 16 routers.

The 4-routers network is our overview example where we considered the same requirements as those described in Section 2.

Protocols / # routers	4 (overview)	9 (Inet2)	16 (grid)
- static routes only	1.8s	18.2s	116.1s
- OSPF and static routes	4.2s	37.0s	197.0s
- OSPF, BGP, and static routes	13.8s	189.4s	577.4s

Table 1: Network-Wide Synthesis time for different protocol combinations and network sizes.

The 9-routers network is Internet2 (Figure 5), a real-world US-based network that connects several major universities and research institutes. The router NEWY is advertising two internal networks (i.e., prefixes) and the router SEAT is advertising one internal network. We required the following path requirements: (i) all internal networks are reachable from any router in the network; (ii) traffic sourced from HOUS and destined to the two internal networks connected beyond NEWY must be load-balanced along HOUS-ATLA-WASH-NEWY and HOUS-KANS-CHIC-NEWY; (iii) traffic sourced from HOUS and destined to any internal networks connected to SEAT must be forwarded along HOUS-KANS-SALT-SEAT.

The 16-routers network corresponds to a 4x4 grid topology. The bottom right router of this network announces one local network. The bottom left and bottom right routers announce an external prefix. We required three path requirements: (i) all routers can reach the local network; (ii) all right-most routers must forward traffic downwards; and (iii) all routers must forward traffic to the bottom right egress router for any external network.

Protocols. We allowed different combinations of protocols: (i) static routes only, meaning that the network forwarding state is determined solely by the static entries; (ii) OSPF and static routes; and (iii) OSPF, BGP, and static routes. The requirements pertaining to internal networks are enforced by all protocol configurations, while those pertaining to external networks are enforced only when BGP is used, as expected.

Results. We run our system on a machine with 16GB of RAM and a modern 4-core processor running at 2.5GHz. The overall synthesis times for the different networks and protocol combinations are shown in Table 1 (median over 10 runs).

The results confirm that our system works in practice, synthesizing a network-wide configuration within 10 minutes in the worst-case. As expected, the synthesis time increases with the network size and the number of protocols involved. We note that the ability of our system to synthesize configurations for ~ 20 nodes means that it is practically useful. Indeed, real networks tend to be hierarchically organized around relatively few regions whose configurations can be synthesized independently before being glued together. As an illustration, out of 232 production networks analyzed by [32], 50% have less than 21 point-of-presences, each

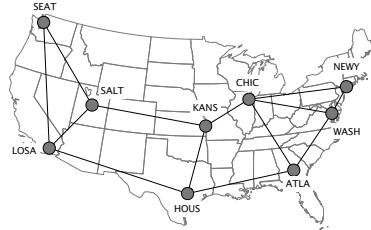


Fig. 5: Internet2 topology

of which could be seen as one (virtual) node. Our system could also be scaled further via additional network-wide constraints to help SMT solving.

8 Related Work

As our work touches on several areas (e.g., Datalog, synthesis, networks, symbolic reasoning), in this section we survey the most closely related representatives from each of these directions.

Analysis of Datalog Programs. Datalog has been successfully used in recent years to specify a variety of scalable static analyzers (e.g., points-to analysis, race detection, and others) with representative works including [33, 34]. Datalog has also been successfully used to model network protocols [4] of the kind that we consider here in the paper. Typically, the focus of these works is on computing a fixed point of the Datalog program on some given initial input (e.g., allocation sites for points-to analysis or a network configuration). In all of these settings, Datalog has proved useful in enabling the designer to declaratively model the essential parts of the analysis/protocol, thus reducing the clutter and overhead which would result if one instead uses a low-level imperative language. Recent work has also focused on extending Datalog to operate with richer classes of lattice structures than the typical powerset lattice [35]. All of these works assume the input is already provided *a priori* and address a rather different problem than we do: our procedure discovers an input that produces a fixed point satisfying a given (user-provided) property on the fixed point.

In terms of computing fixed points, the μZ tool [22] extends the Z3 SMT solver with support for fixed points with logical constraints. Here, given a stratified Datalog program P , an input I , and a constraint φ , μZ checks whether $\llbracket P \rrbracket_I \models \varphi$. To answer this question, the μZ tool first computes the fixed point $\llbracket P \rrbracket_I$ using bottom-up evaluation and then checks the property. However, μZ has no procedure to address our problem. One way is to simply generate different inputs in a brute force manner and run μZ to check if they produce fixed points satisfying the property φ . This naive approach does not work well as we experimentally illustrated in this paper.

The work of Zhang *et al.* [34] presents an algorithm which can be used to efficiently check that certain tuples are not derived for a given set of inputs. Formally, given a Datalog program P (without negation in the literals), a set Q of tuples, and a set \mathcal{I} of inputs, their algorithm computes the set $Q \setminus \bigcap \{\llbracket P \rrbracket_I \mid I \in \mathcal{I}\}$. The algorithm computes this set efficiently by (i) exploiting the monotonicity of P , which guarantees that if a tuple is not derived by a given input I , then it is not derived by any input $I' \subseteq I$, and (ii) using an efficiency pre-order over the inputs \mathcal{I} . The authors apply their algorithm to the problem of automatic abstraction refinement for program analyses specified in Datalog. In this setting, the Datalog program encodes a particular program analysis, and an input I consists of (i) tuples derived from the program under analysis and (ii) tuples that define a particular abstraction. The set \mathcal{I} of inputs consists of all possible abstractions for a given program.

Their algorithm cannot address our problem for two reasons. First, it does not support stratified Datalog programs. Stratified Datalog programs are not monotone and their algorithm relies on monotone programs. Second, although the encoding to SAT can be used to synthesize inputs for each stratum of a stratified Datalog program, it supports only negative properties φ (i.e., properties stipulating that certain tuples must not be derived). The encoding however does not apply to positive properties, which stipulate that certain tuples must be contained in the fixed point. As a result, our approach is more general than [34] and can be used in their application domain as well (as well as other domains such as networks where their approach is not directly applicable).

The FORMULA system [36, 37] can be used to synthesize inputs for non-recursive Dataog programs, as it supports non-recursive Horn clauses with stratified negation (even though [38] which uses FORMULA shows examples of recursive Horn clauses w/o negation). Handling recursion with stratified negation is nontrivial as bounded unrolling is unsound if applied to all strata together. Note that virtually all network specifications require recursive rules, which our system supports. In contrast, FORMULA supports function symbols, which our system does not.

Symbolic Analysis and Synthesis. In spirit, our algorithm is similar to symbolic (or concolic) execution which has gained popularity as a way to automatically and systematically test imperative programs and to generate concrete inputs designating when these programs fail. For an overview of these techniques, see [39]. Generally, these approaches unroll loops up to a bound and try to find inputs (by calling an SMT solver) which violate an assertion (e.g., division by zero) on the symbolic path. In our case, we also find inputs for a symbolic formula, however, the entire setting, the technique using which the formula is built, and the counter-example guided algorithm, are all different than the standard setting of symbolic execution.

Another related line of work is counter-example guided synthesis in which counter-examples are used as a way to prune the search space of candidate programs [40]. However, in synthesis, typically, the goal is to discover a program while in our case, the program is already written and we are trying to find an input satisfying a given property. Nonetheless, there is a connection as in synthesis, sometimes a program can be represented as a vector of bits (i.e., an input that needs to be discovered). However, unlike most synthesis approaches which have a fairly simple one-way direct communication interface between the counter-example generator (i.e., the oracle) and the synthesizer, in our algorithm for generating counter-examples we deal with a sequence of oracles, making the setting more complex and interesting. An interesting future work item is to investigate applications domains where such layered oracle counter-example generation is also applicable and beneficial in terms of improving the efficiency of the search.

Network configuration synthesis. Two previous works consider the problem of synthesizing network-wide configuration: Propane [41] and ConfigAssure [42].

Propane [41] is a recent synthesis framework which also produces network-wide configuration out of high-level forwarding and routing requirements. Unlike our approach though, Propane can only generate configurations encompassing a single protocol, BGP. While this enables Propane to scale better (by limiting the search space), it also reduces the expressiveness of the requirements that it can synthesize. In contrast, our system is strictly more general and expressive as it can synthesize configuration for *any combination* of routing protocols.

ConfigAssure [42] is a general system that takes as input a set of requirements, expressed as first-order constraints, and outputs a configuration that conforms to the input requirements. For example, ConfigAssure can be used to express requirements such as all interfaces must have unique addresses, and it can automatically compute a compliant configuration that assigns addresses to interfaces. The distributed fixed point computation performed by routing protocols, however, cannot be directly captured using the formalism used in ConfigAssure, which lacks fixed point semantics. Therefore, in contrast to our system, ConfigAssure cannot be used to specify networks and, in turn, to synthesize protocol configurations for networks.

9 Conclusion

In this paper, we formulated the problem of network-wide configuration synthesis and presented a general approach to address it. Our approach lets network operators express their global forwarding requirements declaratively before automatically producing a network-wide configuration that will make the routers compute (in a distributed fashion) a compliant forwarding state.

Our key idea is to: *(i)* formally specify the network, i.e. the behavior of all routing protocols and their interactions, as a stratified Datalog program; and to *(ii)* formalize all routing requirements in terms of constraints over the program’s Datalog fixed point, which represents the network’s forwarding state.

The inputs to the resulting Datalog program correspond to actual network-wide configurations, and we can thus construct correct configurations by finding an input to the Datalog program that satisfies the fixed point constraints.

We presented a novel iterative input synthesis algorithm for stratified Datalog, based on decomposing the Datalog rules into strata and iteratively synthesizing inputs for the individual strata using off-the-shelf SMT solvers. We believe that our algorithm for stratified Datalog has applications beyond networks. Based on this algorithm, we have implemented a system that can generate actual configurations for existing routing protocols such as OSPF and BGP. We also presented several experiments that show that automatic network-wide configuration synthesis is a promising alternative to manual network configuration.

Bibliography

- [1] Jenni Ryall. Facebook, Tinder, Instagram suffer widespread issues. <http://mashable.com/2015/01/27/facebook-tinder-instagram-issues/>.
- [2] Juniper Networks. Whats Behind Network Downtime? Proactive Steps to Reduce Human Error and Improve Availability of Networks. Technical report, May 2008.
- [3] BGPMon. Internet prefixes monitoring. <http://www.bgpmon.net/blog/>.
- [4] Ari Fogel, Stanley Fung, Luis Pedrosa, Meg Walraed-Sullivan, Ramesh Govindan, Ratul Mahajan, and Todd Millstein. A general approach to network configuration analysis. In *NSDI*, 2015.
- [5] Nick Feamster and Hari Balakrishnan. Detecting BGP Configuration Faults with Static Analysis. In *NSDI*, 2005.
- [6] Timothy Nelson, Christopher Barratt, Daniel J. Dougherty, Kathi Fisler, and Shriram Krishnamurthi. The margrave tool for firewall analysis. In *LISA*, 2010.
- [7] Lihua Yuan, Hao Chen, Jianning Mai, Chen-Nee Chuah, Zhendong Su, and P. Mopathra. Fireman: a toolkit for firewall modeling and analysis. In *S&P*, 2006.
- [8] Laurent Vanbever, Bruno Quoitin, and Olivier Bonaventure. A hierarchical model for BGP routing policies. In *ACM SIGCOMM workshop on Programmable routers for extensible services of tomorrow*. ACM, 2009.
- [9] X. Chen, M. Mao, and J. Van der Merwe. Pacman: a platform for automated and controlled network operations and configuration management. In *CoNEXT*, 2009.
- [10] William Enck, Thomas Moyer, Patrick McDaniel, Subhabrata Sen, Panagiotis Sedoris, Sylke Spoerel, Albert Greenberg, Yu-Wei Eric Sung, Sanjay Rao, and William Aiello. Configuration management at massive scale: system design and experience. *IEEE Journal on Selected Areas in Communications*, 2009.
- [11] J. Gottlieb, A. Greenberg, J. Rexford, and J. Wang. Automated provisioning of bgp customers. *IEEE Network*, 2003.
- [12] C. Alaettinoglu, C. Villamizar, E. Gerich, D. Kessens, D. Meyer, T. Bates, D. Karrenberg, and M. Terpstra. Routing Policy Specification Language. RFC 2622.
- [13] M. Bjorklund. YANG - A Data Modeling Language for the Network Configuration Protocol (NETCONF). RFC 6020, 2010.
- [14] R. Enns et al. Network Configuration Protocol (NETCONF). RFC 4741, 2011.
- [15] Ahmed El-Hassany, Jeremie Miserez, Pavol Bielik, Laurent Vanbever, and Martin Vechev. Sdnracer: Concurrency analysis for SDNs. In *PLDI'16*.
- [16] Marco Canini, Daniele Venzano, Peter Peresini, Dejan Kostic, Jennifer Rexford, and others. A NICE Way to Test OpenFlow Applications. In *NSDI*, 2012.
- [17] Colin Scott, Andreas Wundsam, Barath Raghavan, Aurojit Panda, Andrew Or, Jefferson Lai, Eugene Huang, Zhi Liu, Ahmed El-Hassany, Sam Whitlock, H.B. Acharya, Kyriakos Zarifis, and Scott Shenker. Troubleshooting Blackbox SDN Control Software with Minimal Causal Sequences. In *ACM SIGCOMM*, 2014.
- [18] Thomas Ball, Nikolaj Bjørner, Aaron Gember, Shachar Itzhaky, Aleksandr Karbyshev, Mooly Sagiv, Michael Schapira, and Asaf Valadarsky. VeriCon: Towards Verifying Controller Programs in Software-defined Networks. In *PLDI*, 2014.
- [19] Sushant Jain, Alok Kumar, Subhasree Mandal, Joon Ong, Leon Poutievski, Arjun Singh, Subbaiah Venkata, Jim Wanderer, Junlan Zhou, Min Zhu, Jon Zolla, Urs Hözle, Stephen Stuart, and Amin Vahdat. B4: Experience with a globally-deployed software defined wan. In *SIGCOMM*, 2013.

- [20] Chi-Yao Hong, Srikanth Kandula, Ratul Mahajan, Ming Zhang, Vijay Gill, Mohan Nanduri, and Roger Wattenhofer. Achieving high utilization with software-driven wan. In *SIGCOMM*, 2013.
- [21] Stefano Vissicchio, Olivier Tilmans, Laurent Vanbever, and Jennifer Rexford. Central Control Over Distributed Routing. In *SIGCOMM*, London, UK, August 2015.
- [22] Kryštof Hoder, Nikolaj Bjørner, and Leonardo De Moura. μZ : An efficient engine for fixed points with constraints. In *CAV*, 2011.
- [23] Shan Shan Huang, Todd Jeffrey Green, and Boon Thau Loo. Datalog and emerging applications: An interactive tutorial. In *SIGMOD*, 2011.
- [24] Alon Y. Halevy, Inderpal Singh Mumick, Yehoshua Sagiv, and Oded Shmueli. Static analysis in datalog extensions. *J. ACM*, 2001.
- [25] Inderpal Singh Mumick and Oded Shmueli. How expressive is stratified aggregation? *Annals of Mathematics and Artificial Intelligence*, 1995.
- [26] Boon Thau Loo, Tyson Condie, Minos Garofalakis, David E. Gay, Joseph M. Hellerstein, Petros Maniatis, Raghu Ramakrishnan, Timothy Roscoe, and Ion Stoica. Declarative networking. *Commun. ACM*, 2009.
- [27] Jeffrey D. Ullman. *Principles of Database and Knowledge-Base Systems*. Computer Science Press, 1989.
- [28] Serge Abiteboul, Richard Hull, and Victor Vianu, editors. *Foundations of Databases: The Logical Level*. 1995.
- [29] <https://logicblox.com/content/docs4/corerefERENCE/html/index.html>.
- [30] C. Barrett, A. Stump, C. Tinelli, S. Boehme, D. d Cok, D. Deharbe, B. Dutertre, P. Fontaine, V. Ganesh, A. to Griggio, J. Grundy, P. Jackson, A. Oliveras, S. Krsti, M. Moskal, L. De Moura, R. Sebastiani, D. Cok, and J. Hoenicke. The SMT-LIB standard: Version 2.0, 2010.
- [31] L. De Moura and N. Bjørner. Z3: An efficient smt solver. In *TACAS'08*.
- [32] Simon Knight, Hung X. Nguyen, Nick Falkner, Rhys Alistair Bowden, and Matthew Roughan. The internet topology zoo. *IEEE Journal on Selected Areas in Communications*, 2011.
- [33] Yannis Smaragdakis and Martin Bravenboer. Using datalog for fast and easy program analysis. In *Datalog Reloaded*, 2010.
- [34] Xin Zhang, Ravi Mangal, Radu Grigore, Mayur Naik, and Hongseok Yang. On abstraction refinement for program analyses in datalog. In *PLDI*, 2014.
- [35] Magnus Madsen, Ming-Ho Yee, and Ondřej Lhoták. From datalog to flix: A declarative language for fixed points on lattices. In *PLDI*, 2016.
- [36] Ethan K. Jackson and Janos Sztipanovits. Towards a formal foundation for domain specific modeling languages. In *EMSOFT*, 2006.
- [37] Ethan K. Jackson and Wolfram Schulte. Model generation for horn logic with stratified negation. In *FORTE*, 2008.
- [38] Ethan K. Jackson, Eunsuk Kang, Markus Dahlweid, Dirk Seifert, and Thomas Santen. Components, platforms and possibilities: Towards generic automation for mda. In *EMSOFT*, 2010.
- [39] Cristian Cadar and Koushik Sen. Symbolic execution for software testing: Three decades later. *Commun. ACM*, 2013.
- [40] Armando Solar-Lezama, Liviu Tancau, Rastislav Bodík, Sanjit Seshia, and Vijay Saraswat. Combinatorial sketching for finite programs. In *ASPLOS*, 2006.
- [41] Ryan Beckett, Ratul Mahajan, Todd Millstein, Jitu Padhye, and David Walker. Don't Mind the Gap: Bridging Network-wide Objectives and Device-level Configurations. In *ACM SIGCOMM*, August 2016.
- [42] Sanjai Narain, Gary Levin, Sharad Malik, and Vikram Kaul. Declarative infrastructure configuration synthesis and debugging. *J. Netw. Syst. Manage.*, 2008.

A Encoding Aggregation using Stratified Datalog

We syntactically extend stratified Datalog with aggregate functions such as `min` and `max`. This extension is possible as stratified Datalog is equally expressive to Datalog with stratified aggregate functions; for details see [25]. To illustrate, consider the aggregate function `min` that appears in the rule $a(X, \text{min}\langle Z \rangle) \leftarrow l(X, Y, Z)$. The semantics of this rule is given by the following two rules

$$\begin{aligned} \text{tmp}(X, Z_2) &\leftarrow l(X, Y_1, Z_1), l(X, Y_2, Z_2), Z_1 < Z_2 \\ a(X, Z) &\leftarrow l(X, Y, Z), \neg \text{tmp}(X, Z) \end{aligned}$$

where `tmp` is a fresh predicate symbol that does not appear in the program. The first rule computes all non-minimal values of Z that appear in the literal $l(X, Y, Z)$ and stores these values in the predicate $\text{tmp}(X, Z)$. The second rule then computes the minimal value of Z , which is the only value that appears in $l(X, Y, Z)$, but not in $\text{tmp}(X, Z)$.

We require that all aggregate functions are used in a stratified way. For our example above, suppose that all rules with a in the head are in a stratum P_i and all rules with l in their heads are in a stratum P_j . Then, we must have $i < j$.

B Encoding Datalog into SMT: Challenges

Given a Datalog program P and a constraint φ , encoding the question $\exists I. \llbracket P \rrbracket_I \models \varphi$ with SMT constraints is non-trivial due to the mismatch between Datalog's program fixed-point semantics and the classical semantics of first-order logic. This means that simply taking the conjunction of all Datalog rules into an SMT solver does not solve our problem. For example, consider the following Datalog program P_{tc} :

$$\begin{aligned} tc(X, Y) &\leftarrow e(X, Y) \\ tc(X, Y) &\leftarrow tc(X, Z), tc(Z, Y) \end{aligned}$$

which computes the transitive closure of the predicate $e(X, Y)$. A naive way of encoding these Datalog rules with SMT constraints:

$$\begin{aligned} \forall X, Y. (e(X, Y) &\Rightarrow tc(X, Y)) \\ \forall X, Y. ((\exists Z. tc(X, Z) \wedge tc(Z, Y)) &\Rightarrow tc(X, Y)) \end{aligned}$$

and we denote the conjunction of these two SMT constraints as $[P_{tc}]$. Now, suppose we have the fixed point constraint $\varphi_{tc} = (\neg e(v_0, v_2)) \wedge tc(v_0, v_2)$ and we want to generate an input I so that $\llbracket P_{tc} \rrbracket_I \models \varphi_{tc}$. A model that satisfies $[P_{tc}] \wedge \varphi_{tc}$ is

$$\mathcal{M} = \{e(v_0, v_1), tc(v_0, v_1), tc(v_0, v_2)\}$$

The input derived from this model, obtained by projecting \mathcal{M} over the *edb* predicate e , is $I_{\mathcal{M}} = \{e(v_0, v_1)\}$. We get

$$\llbracket P_{tc} \rrbracket_{I_{\mathcal{M}}} = \{e(v_0, v_1), tc(v_0, v_1)\}$$

and so $\llbracket P_{tc} \rrbracket_{I_{\mathcal{M}}} \not\models \varphi_{tc}$, which is clearly not what is intended.

$$\begin{array}{c}
\boxed{\varphi \xrightarrow{p} Q} \\
\hline
\frac{}{true \xrightarrow{p} \{p \leftarrow\}} \text{TRUE} \quad \frac{}{a \xrightarrow{p} \{p \leftarrow a\}} \text{PRED} \quad \frac{\overline{X} = fv(\varphi) \quad \varphi \xrightarrow{q} Q}{\neg\varphi \xrightarrow{p} \{p(\overline{X}) \leftarrow \neg q(\overline{X})\} \cup Q} \text{NEG} \\
\\
\frac{\varphi_1 \xrightarrow{q} Q \quad \overline{X}_1 = fv(\varphi_1) \quad \varphi_2 \xrightarrow{r} R \quad \overline{X}_2 = fv(\varphi_2) \quad \overline{X} = \overline{X}_1 \cup \overline{X}_2}{\varphi_1 \wedge \varphi_2 \xrightarrow{p} \{p(\overline{X}) \leftarrow q(\overline{X}_1), r(\overline{X}_2)\} \cup Q \cup R} \text{AND} \\
\\
\frac{\overline{Y} = fv(\varphi) \quad \varphi \xrightarrow{r} Q}{\forall X. \varphi \xrightarrow{p} \{p(\overline{Y}) \leftarrow \neg q(\overline{Y}), q(\overline{Y}) \leftarrow \neg r(\overline{Y} \cup \{X\})\} \cup Q} \text{FORALL}
\end{array}$$

Fig. 6: Translating a constraint φ into a set Q of Datalog rules

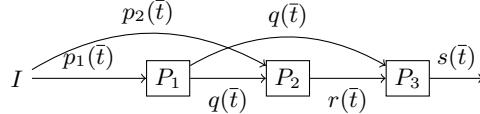


Fig. 7: A Datalog program with strata P_1 , P_2 , and P_3 , and flow of predicates between the strata.

C Translating Constraints into Datalog Rules

In Figure 6, we define a translation from a constraint φ to a set Q of Datalog rules. The rules in Q contain a designated predicate, p , such that for any input I for Q , we have $\llbracket Q \rrbracket_I \models \varphi$ if and only if $p \in \llbracket Q \rrbracket_I$. For example, the rule TRUE maps the constraint $true$ to $Q = \{p \leftarrow\}$. Note that for any input I for Q , we have $\llbracket Q \rrbracket_I \models true$ and $p \in \llbracket Q \rrbracket_I$. The constraint a , where a is a predicate, is mapped to $\{p \leftarrow a\}$. The rules NEG and AND recursively translate the constraints $\neg\varphi$ and $\varphi_1 \wedge \varphi_2$, as expected. Finally, the rule FORALL encodes the constraint $\forall X. \varphi$ with two Datalog rules. Intuitively, since all free variables in the body of any Datalog rule are existentially quantified, we can encode the universal quantifier $\forall X. \varphi$ with the existential quantifier as $\neg(\exists X. \neg\varphi)$. The latter expression is encoded using two rules since we cannot nest the negation operator \neg in Datalog rules.

D Eager Input-Constraint Propagation: Example

We illustrate this point with an example. Consider the Datalog program with strata P_1 , P_2 , and P_3 , depicted in Figure 7. Incoming and outgoing edges of a stratum P_i indicate the *edb* predicates and, respectively, the *idb* predicates of that stratum. For example, the stratum P_3 takes as input predicates $q(\bar{t})$ and $r(\bar{t})$ and derives the predicate $s(\bar{t})$. The algorithm first synthesizes the input I_3 , for the stratum P_3 , which determines the predicates $q(\bar{t})$ and $r(\bar{t})$ that $P_1 \cup P_2$ must output. When generating the constraint ψ_2 (line 10 of algorithm \mathcal{S}_{Strat}),

in addition to constraining the output predicate $r(\bar{t})$, ψ_2 constrains the P_2 's input *edb* predicate $q(\bar{t})$. Suppose we do not constraint the *edb* predicate $q(\bar{t})$ when generating I_2 . Then, the algorithm may end up asking P_1 to output conflicting (i.e., unsatisfiable) constraints for the predicate $q(\bar{t})$ whenever the $q(\bar{t})$ predicates synthesized for I_1 and I_2 do not match.